THICKNESS VARIATION AND NON UNIFORM PLATE BEHAVIOUR USING MODAL TEST RESULTS

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ABSTRACT

This paper presents a proposal to apply model updating to a simple structure, using single measured point to establish dynamic This can be interesting specially for reduced glass thickness because changes of 10% in thickness implies also a 10% on frequencies that could be detected with the proposed procedure. Plates of 1938 x 876 mm, tempered of 10mm and 5mm and annealed laminates 3+3, are selected to obtain free-free natural frequencies, and more than 15 modes are used to update a finite element model in order to study non uniform stiffness distribution due to the production process. Initially, classical modal test approach is used with the measurement of the force and acceleration response on only one point. Finally the possibility of operational modal test is checked using only the response of the structure for more than 15 modes with a measurement on one point. After the updating the FEM model is possible to analyze the similarity with a reference glass plate. This paper presents a first step for the use of Operational Modal Analysis for line quality control of glass plates, during the production process.

Keywords: Operational Modal Analysis, Glass Modal Test, Quality Control, Production line, Safety glasses

1. INTRODUCTION

1.1. General Description

Design and production methods of all kind of manufacturing process have been optimized for years. Almost manufacture plant has a production line testing where every unit produced gets checked as part of its production cycle. Nowadays, quality control is one of the most important stages of manufacturing process.

Real time sounds and vibration analysis are detecting a wide range of mechanical defects and assembly faults in automotive, engine and glass construction industry. This method use the sound and vibration characteristics to identify common defects in the products.

The control of the thickness and flatness tolerances [1] can play an important role. In the production of thin glass solar cells, these kind of quality controls help to reduce the cost of processing defective parts. Typically, thickness, length and width of the glass are inspected at the end of the manufacture, and on delivery. In the case of thin glass with other purposes, geometrical variations could deteriorate the visual quality or change the glass behavior, reaching the fracture.

Most frequent techniques used in the product lines are based on non contact thickness measurement scanning by laser Doppler vibrometers [2], [3], or the measure of the dinamic response in multiple point [4]. All these methods are complex and time-consuming.

Finite element model updating is an inverse problem where experimental data are considered to be exact, and model parameters are updated to reduce differences between the model and the data. Model updating is commonly used in many engineering fields, helping to solve various problems such as structural damage detection, and the estimation of uncertain material parameters. The updating process requires knowledge of dynamic properties; natural frequencies and mode shapes are the most commonly used. Usually first natural frequency or up to four natural frequencies are considered. Measuring response in multiple points is also used frequently. To determine the elastic constants of isotropic materials [5], [6], and to determine elastic and viscoelastic material properties in composite [7], [8], have been studied different procedures.

In this work a proposal to apply model updating to a simple structure, using single measured point to establish dynamic parameters is studied. Complex mathematical procedures like those used for non linear transient phenomena should be applied with an updated model of the structure, which is typical based on modal testing. A fine characterization of a plate with non uniform stiffness properties (E, G) using the Frequency Response Function (FRF) of a single measurement point in a free-free modal test is proposed. Modal parameters obtained with operational modal test are compared with the previous ones in order to simplify the test method for manufacturing process.

The first step in the procedure is to define clearly the structural behaviour of the basic plate, in order to have a reference for comparison with new specimens. For this, a classical modal test is carried out with an adjustment of the FRF and, to avoid an excessive data measurement, a Finite element model is updated and used to identify the modes shapes. Because the structure itself is not complicated, simple geometry and no joints, the frequency and the corner point stiffness for several modes (more than 10) are used for the model updating process.

1.2. Studied Cases

The procedure is applied to tempered monolithic glass plates of 1938 x 876 mm dimensions and 5 and 10 mm thickness, that are slightly anisotropic. As it was mentioned previously, free modal test were carried out; plates were excited with an impact hammer at one of the corners in order to obtain a transfer function with all of the modes of the plate. Acceleration was measured at the same point. Frequency range varies from 0 Hz to 120 Hz.

From the FRF natural frequencies, damping and modal stiffness are estimated. These last parameters allow the identification of the modes of vibration so that mode shapes do not need to be measured in the test. In addition, coupled modes can be identified, which is helpful in the following process. Then an optimization process and non-uniform pattern are established.

Plate FE models use shell element to reproduce plate behaviour with mid-line theory. The orthotropic model presents better results for the glass [9].

2. TEST DATA

2.1. Test and Results

The test data used on this work are part of the experimental work carried out with an impact pendulum designed following the UNE-EN 12600, and is detailed on references [9], [10]. Glass panes of different thicknesses and support conditions have been tested after an identification with two modal test: free-free conditions, used for the present work, and attached to the frame conditions. Figure 1 presents the free-free setup with two accelerometers located on the plate. More details on the test are presented in reference [10].

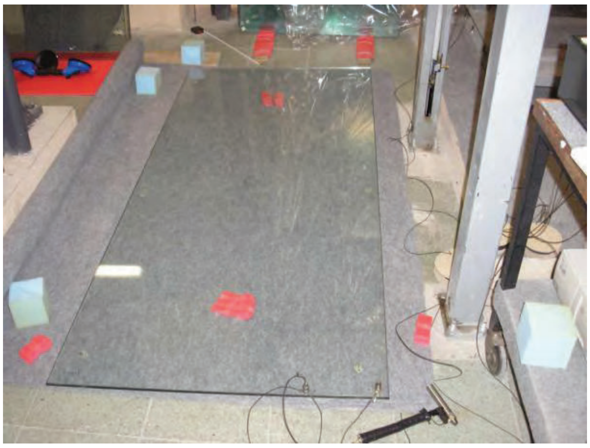

Figure 1. Accelerometer location on free - free condition modal test.

For the current study results of specimens T10, T05, and L33 are used. In order to check for the influence of different stiffness and glass configuration, two different typologies of glass (laminated with PVB, and monolithic), are used. The dimensions are 876 mm width and 1938 mm length with. The main characteristics, the number of plates used for each group, and the number of modal test performed on these typologies are summarized on Table 1.

Table 1. Test specimens selected for the study.

Name	Typology	Thickness (mm)	Number of tested plates	No. Modal tests Free-free
T10	Tempered Monolithic	10	4	6
L33	Annealed Laminated	3 + 0.38 + 3	4	4
T05	Tempered Monolithic	5	4	7

The first data obtained from modal test are the Corner Point Frequency Response Function (CPFRF), because all modes are included on it. Figure 2 plots the CPFRF corresponding with the three specimens. The response of the laminates glass (L33) presents a higher damping.

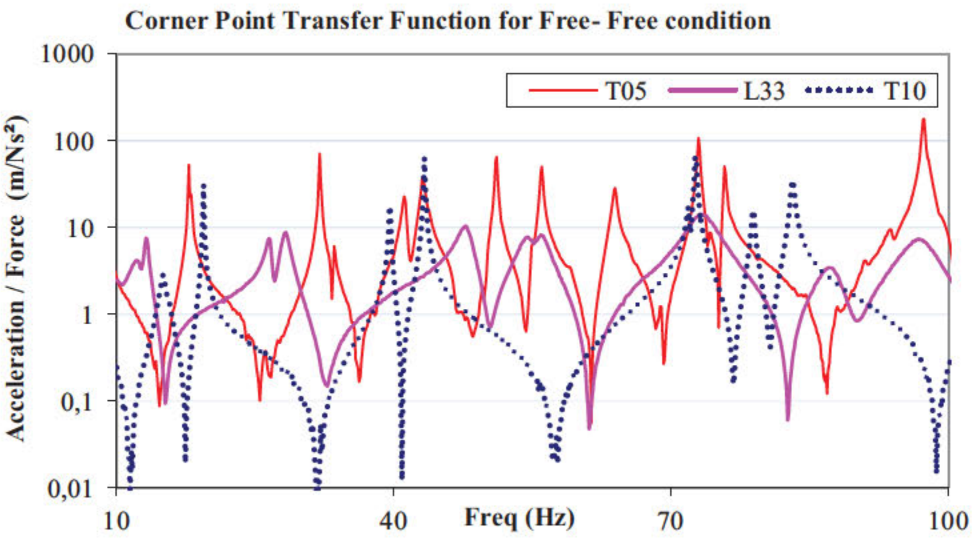

Figure 2. Corner point FRF of each glass plate type.

Another important aspect is the reproducibility of the glass plate response. Figure 3 collects the CPFRF of the four plates tested with L33 typology. It can be seen the high similarity of all the graphs, for resonances and for amplitudes.

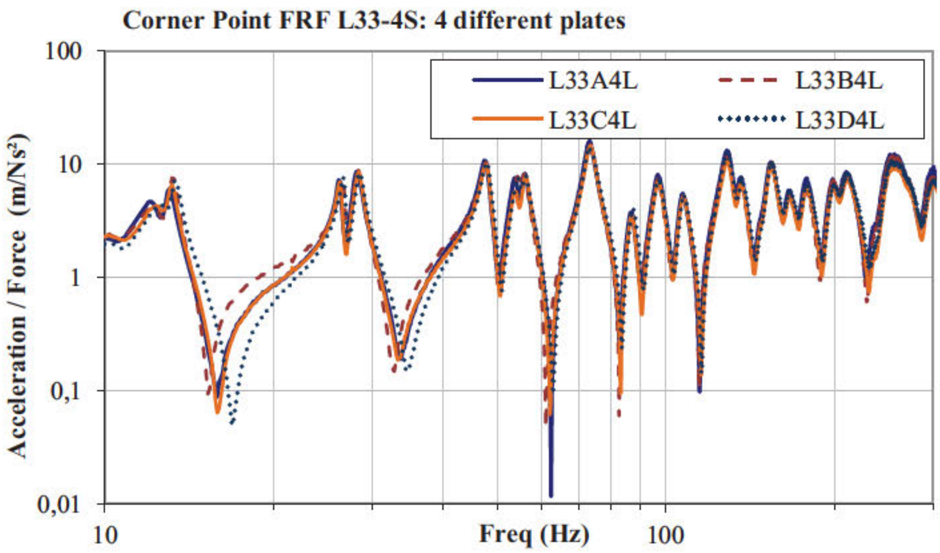

Figure 3. Corner Point FRF (CPFRF) of four different plates for L33 glasses.

3. CLASSICAL MODAL TESTING

3.1. Modal test results

After checking the consistency of the measurements the modal parameters are estimated using a frequency domain procedure implemented in a self developed software, where the response functions are fitted in an iterative process. A general model with complex mode shapes is used allowing out of range modes contributions because the curve fitting is developed in a finite frequency range, and only a limited number of vibration modes are considered.

Equation 1 represents the model used for the frequency response $H_{ij}(w)$:

$$H_{ij}(\omega) = -\frac{1}{m^* \cdot \omega^2} + \frac{1}{i \cdot \omega \cdot c^*} + \sum_{k=m_1}^{m_2} \left(\frac{U_k^{ij} + i \cdot V_k^{ij}}{i \cdot \omega - \alpha_k - i \cdot \Omega_k} + \frac{U_k^{ij} - i \cdot V_k^{ij}}{i \cdot \omega - \alpha_k + i \cdot \Omega_k} \right) + \frac{1}{k^*} \quad (1)$$

where α_k and Ω_k contains information about natural frequencies and damping ratios, U_k^{ij} and V_k^{ij} include the vibration modes information and m^*, c^* and k^* incorporate the influence of the modes not adjusted. The fitting method uses initial values of the desired parameters. These values are obtained using a curve-fit method for each mode, and each degree of freedom. The iterative algorithm modifies the initial parameters in order to minimize the mean quadratic error of every response function simultaneously.

An important information that the fitting process provides is the modal skeleton: modal mass and stiffness obtained with the expressions Eq.(2) and Eq.(3). This skeleton can be used to analyze if the tests and parameters estimation are correct, studying the results symmetry. When force and response is measured in the same point (i=j) it is possible to calculate point modal stiffness which has information on mode shape and natural frequency together and for this reason is used in many fields like damage detection and is incorporated as an important variable in the resent work.

$$m_k^{ij} = \frac{1}{-2 \cdot V_k^{ij} \cdot \omega_{nk} \cdot \sqrt{1 - \zeta_k^2}} \quad (2)$$

$$k_k^{ij} = \frac{\omega_{nk}^2}{-2 \cdot V_k^{ij} \cdot \omega_{nk} \cdot \sqrt{1 - \zeta_k^2}} = \omega_{nk}^2 \cdot m_k^{ij} \quad (3)$$

FRF is generated with estimated parameters and compared with the experimental ones. Figure 4 shows the comparison between experimental and generated FRF for the T05 glass plate. As it can be seen, a very good adjustment is achieved; coherence for T05 glass was 0.992. Similar answer was obtained for the other test; coherence for T10 was 0.991, and for L33 it was 0.944.

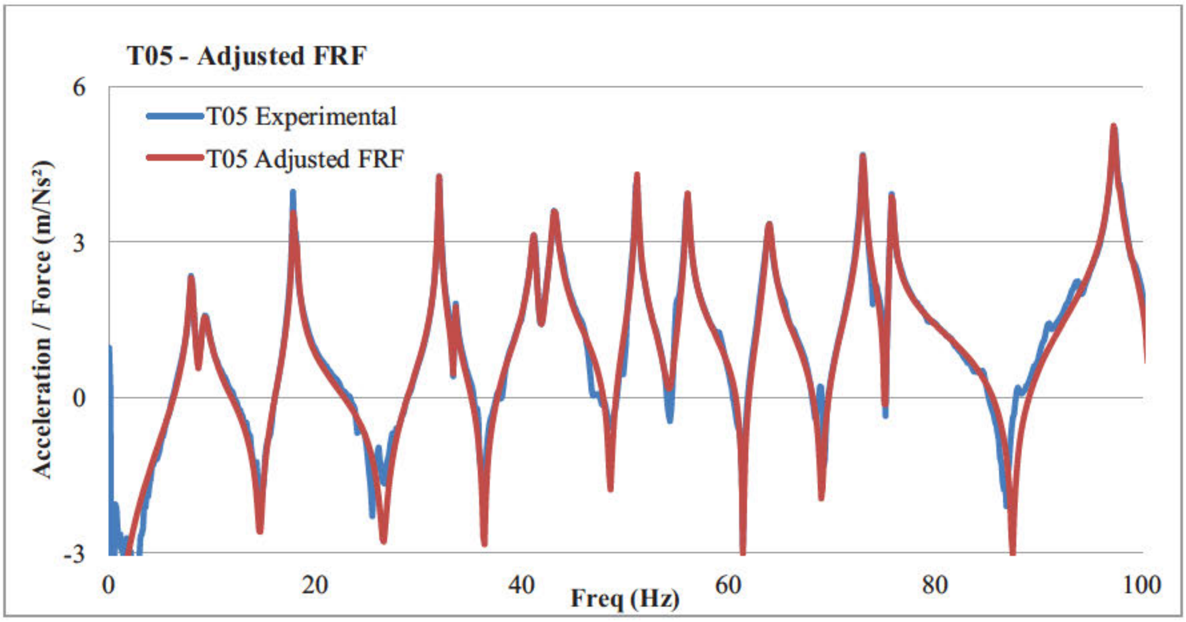

Figure 4: Experimental and adjusted FRF comparison for T05 specimen A.

A total of 12 plates have been tested and adjusted obtaining 35 modes for T05 between 0 and 300Hz, 25 modes for T10 between 0 and 400 Hz, and 15 modes for L33 between 0 and 160 Hz. For each typology the mean values of the frequencies and damping were computed. The standard deviation of the mode frequencies are between 2.5% for T05 plates, and 0.6 for L33. In figure 5 are plotted the first ten frequencies obtained from T05, L33, and T10. In order to check the congruency of the results regarding the thickness of the plate, figure 5 is plotted. X-axes shows 10 natural frequencies of T10, and Y-axes shows the frequencies of the three typologies. It can be observed the influence of thickness as the slopes of the lineal regression presents a good correlation with the actual thickness.

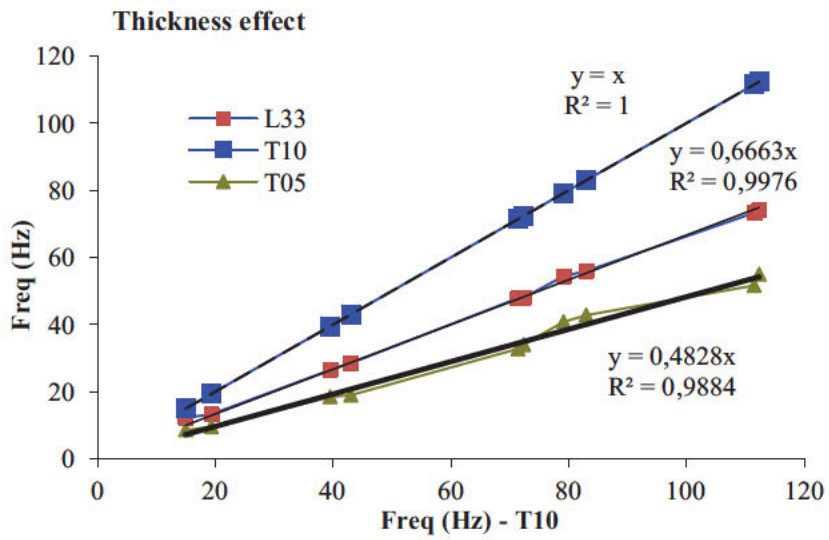

Figure 5: Thickness effect: comparison between natural frequencies.

In Table 2, the mean values of the different plates for the dynamic parameters, natural frequencies, damping and corner point modal stiffness, are collected. For the bending mode double frequencies are detected, showing the good possibilities of the technique used. The damping is clearly higher for laminated glass proving the validity of the results.

3.2. Finite element Model updating

To complete the vibration mode description, their vibration shape is usually needed [5] [6] [11]. The correct identification of mode shapes requires complex and time-consuming methods. However, other authors [12] consider easier parameters, as strain energy, to study mode variations. In this paper, the description of the modes is made using point modal stiffness.

Point modal stiffness, allows a numerical comparison between experimental data and F.E. model. This is useful when modes are coupled. In this situation, natural frequencies are too close and in the F.E. model modes can be wrongly ordered according to their natural frequency. The correct order can be established comparing point modal stiffness.

For the FEM model an element with the mid-plate theory is used, with 60 divisions on each side (31x15 mm) in order to reproduce with enough resolution the mode shape of the higher frequencies. Base on previous studies [9] an orthotropic model for the material have been used. Figure 6 plots four modes shapes.

Table 2. Dynamic parameters estimated for the three types of glasses.

	T05			T10			L33		
Mode	**Freq. (Hz)**	**ζ (%)**	**k_r (kN/m)**	**Freq. (Hz)**	**ζ (%)**	**k_r (kN/m)**	**Freq. (Hz)**	**ζ (%)**	**k_r (kN/m)**
RS	4.66			8.66	5.28	45	10.52	7.08	38
1	8.35	1.93	7	14.78	0.86	92	12.47	4.41	22
2	9.65	3.67	11	19.24	0.20	105	13.23	1.64	39
3	18.21	0.28	44	39.25	0.22	934	26.38	1.09	189
4	32.36	0.33	115	41.94	0.14	465	28.34	1.48	125
5	33.89	0.20	143	70.16	0.17	12600			
6	41.46	0.25	2370	71.60	0.12	1160	48.07	1.56	292
7	43.49	0.46	349	80.22	0.27	3050	54.15	1.14	1350
8	51.24	0.56	186	82.79	0.23	1910	56.43	1.38	659
9	56.17	0.17	414	110.35	0.15	2710	73.53	1.79	393
10	64.00	0.22	548	111.23	0.11	3560			
11	73.13	0.37	800	130.46	0.17	12400	87.03	1.52	2630
12	75.88	0.15	641	149.33	0.12	5220	97.10	1.79	1440
13	97.61	0.14	1590	162.38	0.10	8800	107.56	1.77	1430
14	101.88	0.15	676	197.62	0.12	82400	128.80	2.11	1460
15	108.48	0.15	7710	199.30	0.12	7170			
16	117.04	0.14	2710	207.22	0.12	15000	136.93	1.74	4280
17	124.60	0.14	2170	225.02	0.10	72700			
18	133.66	0.15	1180	236.70	0.11	10600	153.98	1.73	3010
19	147.71	0.23	7600	255.13	0.14	26200			
20	159.14	0.14	2230	273.24	0.11	57200			
21	160.64	0.10	37900	274.29	0.11	30100			
22	177.55	0.12	2110	305.76	0.10	22800			
23	191.05	0.16	7810	323.32	0.12	39200			
24	191.49	0.12	7260	327.62	0.12	53500			
25	200.59	0.12	9570	363.10	0.11	114000			
26	208.03	0.12	2990						
27	228.07	0.13	6930						
28	230.99	0.18	41700						
29	241.89	0.08	46100						
30	243.38	0.17	6550						
31	254.93	0.14	12100						
32	269.98	0.12	110000						
33	287.25	0.11	8920						
34	295.90	0.11	6510						
35	298.71	0.10	13500						

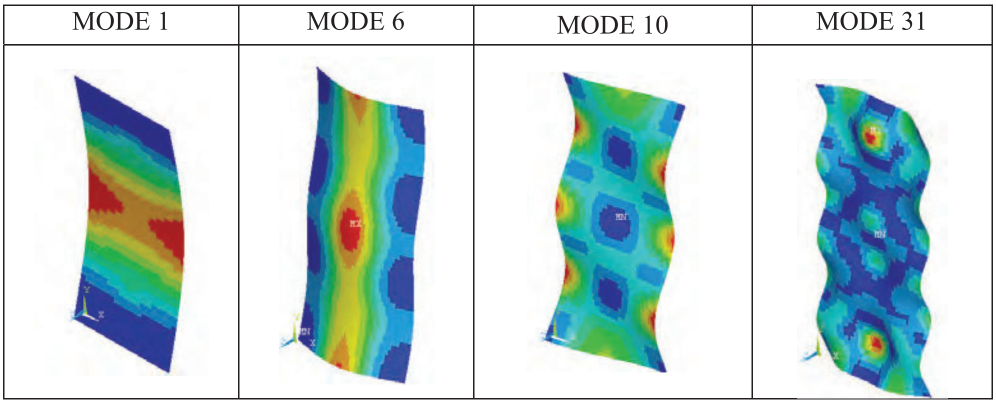

Figure 6: Some Mode Shapes of the plate.

In order to adjust the FEM with the test data for the frequencies, an optimization procedure has-been used.

The objective function used for the model updating is the root mean square (RMS) of the frequencies differences Eq.(4), and the design variables are the Young Modulus for two directions. Table 3 presents the results of optimizations performed for the three glasses. The first column indicates the type of glass. The second and third columns are the updated values for the Young's Modulus of each model. The last column represents error in frequencies between experimental and model data.

$$RMS = \sqrt{\frac{\sum_1^N \left(\frac{f_{test,i}\ \ f_{model,i}}{f_{test,i}}\right)^2}{N}} \tag{4}$$

Table 3. Optimization process results for the glass plates.

	Young's modulus (GPa)		**Frequency error**
Model	**E_x**	**E_y**	**RMS (%)**
T10	67.8	70.0	4.19
T05	68.7	69.4	4.66
L33	71.7	73.6	1.01

4. OPERATIONAL MODAL ANALISYS

4.1. Operational Modal procedure

After determining the modal parameters of the frequency response for the considered plates by using the Classical Modal Analysis techniques, the next step consists on obtaining similar results by using an alternative method. The Operational Modal Analysis techniques are a good alternative, since it is necessary to measure only the response of the structure. The Stochastic Subspace Identification (SSI) methods are one of the most robust output-only identification techniques. Those methods are based on complex mathematical algorithms where the modes are extracted directly from time data samples.

Consequently, to apply this technique it has been necessary to develop a software tool able to compute all these complex mathematics in a fast way. This issue has been solved by using National Instruments LabView 9, which provides a library of Modal Analysis tools including the mentioned algorithm.

To better understand the way the modes are obtained, the entire implemented method is shown step-by-step as follow:

1. Once the time sample containing the time-domain response of the plate to study has been selected, it is necessary to filter this signal in order to remove noise and frequency components that are out of the range of interest. To do that, each signal has been filtered by using a Butterworth Band-pass Filter (order 10).

2. Then, the number of modes to find within the frequency range, determined by the lower and the upper cut-off frequencies of the bandpass filter must be specified.

3. To finally obtain the modal information an iterative routine is implemented. The SSI algorithm is applied and once the computed modes are obtained, those ones with a negative or excessive (>10%) damping are removed. So, the method is applied continuously considering these restrictions until the number of obtained modes is equal to the number of modes to find, as it was specified in the previous step. All them are within the established frequency and damping limits.

4. After that, the tested FRF for CM can be compared with the OMA in frequency response function. Figure 6 plots both graphs. A great coincidence for the resonant zones is obtained

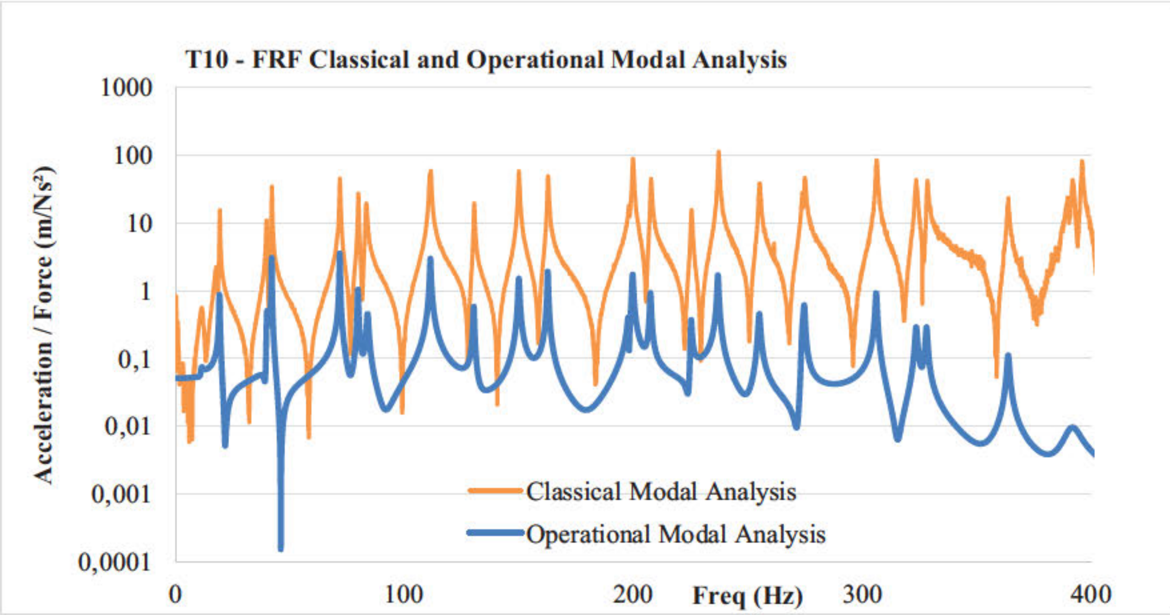

Figure 6: Comparison of FRF obtained with Operational and clasical Modal Analysis.

4.2. OMA final adjustment of plates

Following the proposed procedure, the adjustment of the tree plates is done. Table 4 collects all the frequencies compared with the values from classical modal test. it is observed that due to the higher damping the number of modes obtained for the L33 is lower. The correlation is very good. Two aspects can be observed: the number of loose modes (4 from 35), and the good approach of the frequencies for the detected modes. This can be observed representing both groups of frequencies on X and Y axis (figure 7) and the regression line gives the approximation with the slope (similar to MSF) and the regression coefficient (similar to MAC for mode shapes). On table 5 are reflected the values obtained for the correlation between frequencies obtained from classical and operational modal analysis: regression line for T05 L33 and T10 plates.

Table 4. Comparison between CM and OMA experimental frequencies.

Frequency (Hz)														
T05						T10						L33		
nº	CM	OMA	nº	CM	OMA	nº	CM	OMA	nº	CM	OMA	nº	CM	OMA
1	4.7		19	133.7	133.4	1	8.7		19	236.7	236.8	1	10.5	
2	8.4	8.1	20	147.7	147.4	2	14.8	11.2	20	255.1	254.8	2	12.5	13.0
3	9.7		21	159.1	158.4	3	19.2	19.1	21	273.2	273.5	3	13.2	
4	18.2	17.9	22	160.6	160.4	4	39.3	39.8	22	274.3	274.5	4	26.4	24.9
5	32.4	32.0	23	177.6	177.4	5	41.9	41.9	23	305.8	305.9	5	28.3	28.3
6	33.9		24	191.1	190.3	6	70.2	70.5	24	323.3	323.4	6	48.1	47.8
7	41.5	41.1	25	191.5	190.8	7	71.6	71.6	25	327.6	327.9	7	54.2	54.1
8	43.5	43.2	26	200.6	200.5	8	80.2	79.6	26	363.1	363.6	8	56.4	57.0
9	51.2	51.1	27	208.0	208.2	9	82.8	83.8	27		239.9	9	73.5	73.2
10	56.2	56.0	28	228.1	228.0	10	110.4	111.2				10	87.0	86.9
11	64.0	63.9	29	231.0	230.8	11	111.2	111.5				11	97.1	96.9
12	73.1	73.0	30	241.9	241.9	12	130.5	130.2				12	107.6	107.1
13	75.9	75.8	31	243.4	243.5	13	149.3	149.8				13	128.8	129.0
14	97.6	97.4	32	254.9	254.9	14	162.4	162.5				14	136.9	135.8
15	101.9	101.6	33	270.0	269.9	15	197.6	197.7	Rigid Solid Modes			15	154.0	153.9
16	108.5	108.2	34	287.3	287.2	16	199.3	199.5	Disappeared modes			16		115.2
17	117.0	116.8			94.3	17	207.2	207.3	Wrong modes			17		146.1
18	124.6	124.2			145.7	18	225.0	225.1				18		

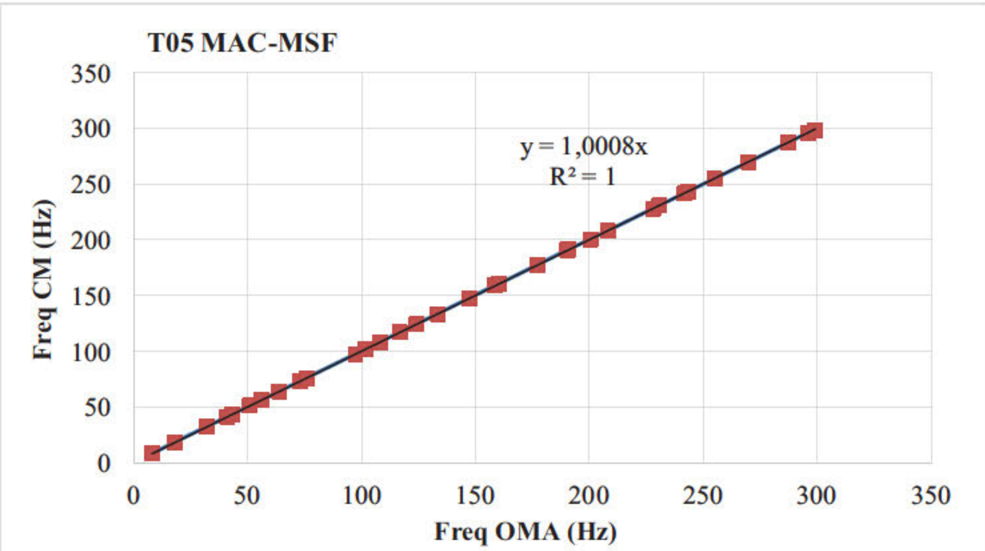

Figure 7. T05 Comparison of frequencies (Hz) obtained with Clasical and Operationa Modal test (30 modes).

Table 5. Correlation between CM and OMA: Regression line

Type of Glass	Nº Disappeared Modes	Nº wrong Modes	R^2	slope
T05	2	2	1	1.0008
T10	0	1	0.9999	0.9993
L33	1	2	0.9998	1.0027

5. CONCLUSIONS

This paper proposes the first step of a method for measuring the thickness of a glass plate based on the use of modal stiffness at one single point.

The use of modal stiffness at one single point with FE model, is appealing as it can be easily used in simple structure models updating and using data from non-destructive modal test.

The natural frequencies of the plate changes in the same way that the thickness. This fact allows knowing geometrical differences between the test plate and the reference one.

Operational Modal analysis has been used to estimate the dynamic response of free-supported system. With operational modal test the natural frequencies can be determined as well as from the classical way. It can be said that operational modal analysis can be implemented as quality assurance.

Modal stiffness at one point provides all the information needed for the structural behavior of the plate. This fact allows detecting non uniform stiffness distributions due to manufacturing process.

Therefore, with this method, thickness and other manufacturing errors can be detected. This measuring procedure avoids the typical problems of excessive time consuming or the high cost of some measurement systems. For all these reasons, a more detailed study should be performed to incorporate this alternative methodology in the current quality control process.

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